Manufacturing cost minimisation through scrap and rework modelling

UTC for Computational Engineering
Christopher Dodd, Prof. James Scanlan, Dr. Robert Marsh
Faculty of Engineering and the Environment
Dr. Steve Wiseall, Rolls-Royce plc.

Introduction
The broad scope of this research is cost optimisation. In particular this project focusses on the generation of scrap and rework in the manufacture process. It can be shown in some cases manufacturing cost can be reduced by shifting the nominal value of a design parameter to bias rework opposed to scrap. However, this impacts the performance of the component generating a trade-off between cost and performance.

Outline of the problem
Robust Design can be characterised by designing products to be insensitive to variation. In a manufacturing context, components can be robustly designed against differences in performance due to variation in the manufacturing process. Parameter design is typically implemented to achieve this. With knowledge of the likely variation in the design parameters (due to manufacturing variation) the aim of parameter design is to set the nominal values of the design parameters such that the number of conforming parts (conforming to performance specification limits) is maximised. However, during manufacture scrap and rework may still be produced if dimensions of the manufactured component lie outside the tolerance limits set to ensure the desired performance is achieved. Since the cost of scrap is generally greater than rework cost, a minimum cost point will exist close to the nominal design point given from parameter design, but shifted to bias rework on the design parameters. This is generally referred to an optimal mean setting.

Figure 1 indicates this for a one-dimensional example. The solid green line represents the distribution of the design variable centred on the nominal value which lies in the centre of the tolerance range (lower and upper specification limits). Both scrap and rework are generated. However, scrap can be minimised by shifting the distribution right (as indicated by the dotted green line). The new nominal still lies between the tolerance limits but the cost of manufacturing the components will be less if the scrap cost is significantly greater than the rework cost.

Example
Consider the component shown in Figure 2. There are four critical features requiring inspection, the outer diameters 1 and 2 and the wall thicknesses 1 and 2. It should be noted Thickness 1 = Diameter 1 – Diameter 3 and similarly Thickness 2 = Diameter 1 – Diameter 4. It is assumed the outer diameters are created through a turning operation on a lathe while the inner diameters are created with a boring operation. The challenge was to determine the optimal mean settings to maximise overall profit as well as return the manufactured distribution of the geometry.

Method and Results
As can be seen from Figure 1, biasing the means creates rework which must undergo re-machining. An absorbing Markov chain was used to determine the total cost of rework and it was found a truncated normal distribution could be used to model the final distribution of the outer diameters. The Thickness 1 and Thickness 2 distributions were found from the convolution of the Diameter 1 and Diameters 3 and 4 in the form $T = D_{outer} + (-D_{inner})$ where the distribution was given by,

$$T(d_o) = \int_{-\infty}^{\infty} f_{D_{outer}}(d_0) f_i(d_o - d_i) \, dd_i.$$

The subscripts $o$ and $i$ indicate the outer and inner diameters and $f_i$ represents the distribution function of the outer diameters. A bivariate distribution used created to determine the initial scrap and rework probabilities from the thickness parameters using a copula function of the form $F(t_1, t_2) = C(\Phi_{T_1}(t_1), \Phi_{T_2}(t_2))$. The term $\Phi_{T_1}(t_1)$ was the marginal cumulative distribution of the Thickness 1 parameter and similarly for the Thickness 2 parameter. The means of the four diameters were optimised to maximise an arbitrary profit function where scrap cost > rework cost. The final distributions for the inspected parameters are given below.

Conclusion
Figure 3 clearly indicates how far the manufactured geometry distributions are from a normal distribution. This has a significant impact of average part mass and will also influence average component performance. Process improvement strategies and six-sigma philosophies aim to ensure production is always capable, eliminating scrap and rework. However, the optimal mean setting practice discussed here allows the impact of tightening tolerance beyond process capability to be considered, and the use of copula statistics allows this analysis to be performed at the design stage.

Acknowledgements
This work is conducted through a Rolls-Royce plc. led research programme SILOET (Strategic Investment in Low Carbon Engine Technology). Funding is provided by Rolls-Royce, TSB (Technology Strategy Board) and the EPSRC (Engineering and Physical Sciences Research Council).